

## 512. MATHEMATICS

### Algebra

Automorphisms- Conjugacy and G-sets- Normal series solvable groups- Nilpotent groups. Structure theorems of groups: Direct product- Finitely generated abelian groups- Invariants of a finite abelian group- Sylow's theorems- Groups of orders  $p^2$ , Ideals and homomorphism- Sum and direct sum of ideals, Maximal and prime ideals- Nilpotent and nil ideals- Zorn's lemma

Unique factorization domains - Principal ideal domains- Euclidean domains- Polynomial rings over UFD- Rings of fractions.

### Real Analysis

Metric spaces- Compact sets- Perfect sets- Connected sets

Limits of functions- Continuous functions- Continuity and compactness Continuity and connectedness- Discontinuities – Monotone functions.

Riemann- Stieltjes integral- Definition and Existence of the Integral- Properties of the integral- Integration of vector valued functions- Rectifiable curves.

Sequences and series of functions: Uniform convergence- Uniform convergence and continuity- Uniform convergence and integration- Uniform convergence and differentiation- Approximation of a continuous function by a sequence of polynomials.

### Functional Analysis

Normed Space, Banach Space, further properties of normed spaces, Finite dimensional normed spaces and subspaces, compactness and finite dimension linear operators, Bounded and continuous linear operators, linear functionals, linear operators and functionals on finite dimensional spaces, normed spaces of operators, Dual spaces.

Inner product space, Hilbert space, further properties of inner product spaces, orthogonal complements and direct sums, orthonormal sets and sequences, series related to orthonormal sequences and sets.

Total Orthonormal sets and sequences, Representation of functionals on Hilbert Spaces, Hilbert-adjoint operator, self-adjoint, unitary and normal operators.

Hahn-Banach theorems for Complex vector spaces and normed spaces, adjoint operator, Reflexive spaces, uniform boundedness theorem, convergence of sequences of operators and Functionals. Open mapping theorem, closed graph theorem.

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## Complex Analysis

Regions in the complex plane- Functions of a complex variable- Mappings by exponential functions- Limits- Continuity- Derivatives- Cauchy-Riemans equations- Sufficient conditions for differentiation- Polar coordinates.

Analytic functions- Uniquely determined analytic functions- Reflection principle- The exponential function- The logarithmic function- Complex exponents- Trigonometric functions- Hyperbolic functions- Inverse trigonometric- Hyperbolic functions.

Derivatives of functions  $w(t)$ - Definite integrals of functions  $w(t)$ - Contours- Contour integrals- Upper bounds for moduli of contour integrals- Anti derivatives.

Cauchy-Goursat theorem and its proof- Simply and multiply connected domains- Cauchy's integral formula- Derivatives of analytic functions- Liouville's theorem and fundamental theorem of algebra- Maximum modulus principle.

## Mathematical Methods

Existence and Uniqueness of solution of  $\frac{dy}{dx} = f(x,y)$ . The method of successive approximation- Picard's theorem- Sturm-Liouville's boundary value problem.

Partial Differential Equations: Origins of first-order PDES-Linear equation of first-order-Lagrange's method of solving PDE of  $P_p+Qq = R$  – Non-Linear PDE of order one-Charpit method- Linear PDES with constant coefficients.

Partial Differential Equations of order two with variable coefficients- Canonical form Classification of second order PDE- separation of variable method solving the one-dimensional Heat equation and Wave equation- Laplace equation.

Power Series solution of O.D.E. – Ordinary and Singular points- Series solution about an ordinary point -Series solution about Singular point-Frobenius Method.

Legendre Polynomials: Legendre's equation and its solution- Legendre Polynomial and its properties- Generating function-Orthogonal properties- Recurrence relations- Laplace's definite integrals for  $P_n(x)$ - Rodrigue's formula.

Bessels Functions: Bessel's equation and its solution- Bessel function of the first kind and its properties- Recurrence Relations- Generating function- Orthogonality properties.

Hermite Polynomials: Hermite's equation and its solution- Hermite polynomial and its properties- Generating function- Alternative expressions (Rodrigue's formula)- Orthogonality properties- Recurrence Relations.