516. STATISTICS

1. **Probability :** Sample space, events, relations among events, classical and relative frequency definitions of probability, probability as a measure. Basic results on probability of events. Conditional probability and Baye's theorem. Independence of events.

   Random variables (discrete and continuous). Distribution function and its properties. Joint distribution of two and more random variables. Marginal, conditional distributions and densities. Expectation of random variables, moments and generating functions. Conditional expectation. Characteristics function and its properties. Inversion theorem. Statement of continuity theorem.

   Convergence of a sequence of events. Borel – Cantelli lemma, Borel 0-1 law and statement of Kolmogorov 0-1 law with applications. Convergence of a sequence of random variables. Convergence in law, in probability, with probability one and in quadratic mean and other inter-relationships. Convergence in law of $X_n + Y_n$, X_nY_n and X_n/Y_n. Definition and examples of weak law of large numbers. Khintchene's theorem and strong law of large numbers.

   Statement of CLT. Lindberg-Levy and Liapunov forms of central limit theorems, statement of Lindberg – Feller form of CLT with simple illustrations.

   Stochastic processes with examples. Markov Chains transition probability matrix and classification of states of a Markov chain with examples.

2. **Distribution Theory :** Theoretical distribution – Binomial, Poisson, negative binomial, geometric, hypergeometric, multinominal, rectangular, normal, lognormal, exponential, gamma, beta, Cauchy, weibull and Pareto distributions with properties.

   Transformation of random variables. Distribution of Chi – squares, t and F distributions and their properties. Distribution of $\overline{X}$ and s^2 for samples coming from normal population. Distribution of order statistics and range. Joint and marginal distribution of order statistics. Distribution of sample quantiles.

   Multivariate normal distribution and its marginal and conditional distribution with examples. Simple correlation and lines of regression.

3. **Estimation :** Unbiasedness, sufficiency, consistency and efficiency of a point estimate with examples. Statement of Neyman's factorization criterion with applications. Minimum variance unbiased estimation, Crammer – Rao lower bound and its applications. Rao – Blackwell theorem, completeness and Lehman – Scheffe theorem. Estimation by method of maximum likelihood, moments and statement of its properties. Confidence intervals for the parameters of normal, exponential, binomial and Poisson distribution.

4. **Testing of Hypotheses :** Concepts of tests of statistical hypothesis, types of error, level of significances, power, critical region and test function. Concepts of MP and UMP tests. Neyman – Pearson lemma and its applications, one parameter exponential family of distributions. Concepts of unbiased and consistent tests. Likelihood ratio (LR) criterion with simple applications (including homogeneity of variances). Statements of asymptotic properties of LR tests. Large sample tests of population means, proportions and correlation coefficients. Relation between confidence intervals, and hypothesis testing. Wald's SPRT for testing a simple null hypothesis against simple alternative hypothesis and its OC and ASN functions. SPRT procedure for binomial, Poisson, normal and exponential distributions.

5. **Non – Parametric Tests :** Non – parametric tests for (i) one sample case: sign test, Wilcoxon signed rank test for symmetry, runs test for randomness, Kolmogorov – Smirnov (k-s) test for goodness of fit (ii) two sample case: sign and Wilcoxon tests for paired comparisons. Wilcoxon - Mann Whitney test

and K –S test and test for independence based on spearman's rank correlation. Kruskal-Wallis test and Friedman's test.

6. **Multivariate Tests :** Principal Component Analysis, Factor analysis, Canonical Correlation, Cluster analysis. Multivariate tests based on Hotelling's T^2 and Mahalanobis D^2 statistics for one sample problem, two sample problem and classificatory problems between two normal populations based on Fisher's discriminant function.

7. **Sampling Techniques :** Estimation of population mean, population total and variance of the estimator in the following sampling methods: simple random sampling with and without replacements and equal and unequal probabilities. Horwitz Thompson and Yates and Grundy estimators. Selection of sample and determination of sample size. Stratified random sampling, proportional and optimum allocations and comparisons. Systematic sampling with N=nk and comparisons in populations with linear trend. Cluster sampling with clusters of equal and unequal sizes. Two stage sampling with equal and unequal first stage units. Ratio and regression estimation in case of simple random sampling and stratified random sampling. Non – sampling errors.

8. **Linear Models and Analysis of Experimental Designs:** Gauss – Markov linear model, BLUE for linear functions of parameters Gauss – Markov theorem, analysis of multiple regression models, multiple and partial correlations. Tests of hypothesis on regression and correlation parameters, tests of sub – hypothesis. Aitken's generalized least squares. Concept of multicollinearity.

   Introduction of selecting the best regression equation, all possible regressions: backward, stepwise regression procedures. Variations on these methods. Ridge and robust regression procedures. Probit and logit analysis, Introduction to non-linear regression model building, least squares in non-linear case, estimating the parameters, non-linear growth models.

   Statement of Cochran's theorem for quadratic forms, analysis of variance one – way classification model, two – way classification model with one - observation per cell with more than one (equal) observations per cell with interaction. Fisher's least significance difference (LSD) method. Analysis of covariance one-way and two – way classification. Fundamental principles of experimental designs. Analysis of completely randomized design (CRD), Randomized Block Design (RBD), and Latin Square design (LSD). Analysis of RBD and LSD with one and more than one observation missing.

   Estimation of main effects, interactions and analysis of 2^2, 2^3, 2^4, 2^n and 3^2 factorial experiments. Total and partial confounding of 2^2, 2^3, 2^4 and 3^2 factorial designs. Concept of balanced partial confounding. Fractional factorial designs. Split plot design and its analysis.

   Balanced incomplete block design (BIBD) - parametric relations, Intra – block analysis and recovery of inter block information. Partially balanced incomplete block design with two associate classes (PBIBD (2)) – parametric relations and intra –block analysis. Youden Square design, Lattice design and intra – block analysis of simple lattice design.

9. **Optimization Techniques :** Meaning and scope of Operations research, formulation of Linear programming problem (LPP), rule of steepest ascent, and θ-rule, optimum solution for Linear programming problem by graphical method and simplex algorithm using artificial variables (Big M/penalty method and two phase simplex methods). Dual of a symmetric Linear programming problem and reading the optimal solution to the dual from the optimum simplex table of primal. Complementary slackness theorem, dual simplex algorithm.

   Definition of transportation problem, initial basic feasible solution by north west, matrix minimum methods and VAM. Optimal solution through MODI tableau for balanced and unbalanced transportation problem, degeneracy in transportation problem, transportation problems as a special case of linear programming problem. Assignment problem as a special case of transportation problem and LPP. Optimal solution using Hungarian method.

Sequencing: Optimal sequence of ‘n’ jobs on two and three machines without passing.

Non–linear programming problem – Formulation, generalized Lagrange multiplier technique, Kuhn - Tucker necessary and sufficient conditions for optimality of an NLPP.

Game theory: 2 person zero sum game, pure strategies with saddle point, principles of dominance and games without saddle point.

Introduction to simulation, generation of random numbers for uniform, Normal, Exponential, Cauchy and Poisson distributions. Estimating the reliability of the random numbers, simulation to queuing and inventory problem.

Queuing Theory: Introduction, essential features of Queuing system, Operating characteristics of Queuing system (transient and steady states). Queue length, General relationships among characteristics. Probability distribution in queuing systems, distribution of Arrival and inter arrival. Distribution of death (departure) process, service time .Classification of Queuing models and solution of Queuing models; M/M/1: ∞/FIFO and M/M/1: N/FIFO.
Integer Programming Problem: Gomory’s cutting plane algorithm for pure and mixed IPP; Branch and bound Technique.